## Matrix multiplication:-

The product AB of a row matrix $A = [a_i]$ and a column matrix $B = [b_i]$ with the same number of elements is defined to be the scalar obtained by multiplying corresponding entries and adding, i.e

$$AB = [a_1, a_2, \ldots, a_n] \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$= a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$

$$= \sum_{k=1}^{n} a_k b_k$$

### Example

a) $[7, -4, 5] \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} = 7\times3 + -4\times2 + 5\times-1$

$= 21 - 8 - 5$

$= 8$

b) $[6, -1, 8, 3] \begin{bmatrix} 4 \\ -9 \\ -2 \\ 5 \end{bmatrix} = 24 + 9 - 16 + 15$

$= 34$

**Definition:** Suppose $A = [a_{ik}]$ and $B = [b_{kj}]$ are matrix such that the number of columns of A is equal to the number of rows of B, say, A is an $m\times p$ matrix and B is a $p\times n$ matrix, Then the Product AB is the $m\times n$ matrix whose ij entry is obtained by multiplying the ith row of A & jth column of of B.

$$\begin{bmatrix} a_{11} & \cdots & a_{1p} \\ \cdots & \cdots & \cdots \\ a_{i1} & \cdots & a_{ip} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mp} \end{bmatrix} \begin{bmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \cdots & & \cdots & & \cdots \\ b_{p1} & \cdots & b_{pj} & \cdots & b_{pn} \end{bmatrix}$$

$$= \begin{bmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & c_{ij} & \vdots \\ c_{m1} & \cdots & c_{mn} \end{bmatrix}$$

where, $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{ip}b_{pj}$

$$= \sum_{k=1}^{p} a_{ik} b_{kj}$$

Exmple:

a) find AB where, $A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

$B = \begin{bmatrix} 2 & 0 & -4 \\ 5 & -2 & 6 \end{bmatrix}$

$\therefore AB = \begin{bmatrix} 2+15, & 0-6, & -4+18 \\ 4-5, & 0+2 & -8-6 \end{bmatrix}$

$= \begin{bmatrix} 17 & -6 & 14 \\ -1 & 2 & -14 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 0 & -2 \end{bmatrix}$

find AB ?

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## Transpose of a matrix

The transpose of a matrix $A$ is $A^T$, is the matrix obtained by writing the columns of $A$, ~~in other~~

**Example:**

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

and $$[1, -3, -5]^T = \begin{bmatrix} 1 \\ -3 \\ -5 \end{bmatrix}$$

**Theorem:-**

i) $(A+B)^T = A^T + B^T$

ii) $(A^T)^T = A$

iii) $(KA)^T = KA^T$

iv) $(AB)^T = B^T A^T$

## Square matrix:-

A square matrix is a matrix with the same number of rows as columns; An $n \times n$ square matrix is said to be of order $n$ and is sometime called an $n$-square matrix.

**Example:**

square matrix of order 3

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & -4 & -4 \\ 5 & 6 & 7 \end{bmatrix} \quad \& \quad B = \begin{bmatrix} 2 & -5 & 1 \\ 0 & 3 & -2 \\ 1 & 2 & -4 \end{bmatrix}$$

## Diagonal and Trace

Let $A = [a_{ij}]$ be an n-square matrix. The diagonal or main diagonal of A consists of the elements with the same subscripts i.e

$$a_{11}, a_{22}, a_{33}, ---, a_{nn}$$

The trace of A, written $tr(A)$, is the diagonal elements

$$tr(A) = a_{11} + a_{22} + a_{33} + --- + a_{nn}.$$

Theorem:-

i) $tr(A+B) = tr(A) + tr(B)$

ii) $tr(KA) = K\, tr(A)$

iii) $tr(A^T) = tr(A)$

iv) $tr(AB) = tr(BA)$

Let, A and B be the matrices. ~~from~~

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & -4 & -4 \\ 5 & 6 & 7 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & -5 & 1 \\ 0 & 3 & -2 \\ 1 & 2 & -4 \end{bmatrix}$$

diagonal of A = $\{1, -4, 7\}$

diagonal of B = $\{2, 3, -4\}$

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$\therefore tr(A) = 1-4+7 = 4$

$tr B = 2+3-4 = 1$

$tr(A+B) = 3-1+3 = 5$

$tr(2A) = 2-8+14 = 8$

$tr(A^T) = 1-4+7 = 4$

$tr(AB) = 5+0-35$

$= 30$

$tr(BA) = 27-24-33$

$= -30$

$\therefore tr(A+B) = tr(A) + tr(B)$

$tr(A^T) = tr(A)$

$tr(2A) = 2\, tr(A)$

Identity matrix, Scalar matrices

The n square identity or unit matrix, denoted by $I_n$, or simply $I$, is the n square matrix with 1's on the diagonal and 0's elsewhere. The identity matrix $I$ is similar to the scalar 1 in that for any n square matrix A.

$$AI = IA = A$$

$$(KI)A = K(IA) = KA$$

example: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$, $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

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